Tertiary Education Commission
Te Amorangi Mātauranga Matua

TEACHING ADULTS TO MEASURE AND INTERPRET SHAPE AND SPACE

USING THE LEARNING PROGRESSIONS

Mā te mōhio ka ora:
mā te ora ka mōhio

Through learning there is life:
through life there is learning!

The Tertiary Education Commission would like to thank the many people who have contributed their time and expertise to this document, in preparation, design and development, consultation and review.

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Introduction

Teaching Adults to Measure and Interpret Shape and Space: Using the Learning Progressions is part of a set of resources developed to support the teaching of literacy, language and numeracy for adult learners. The end goal is to enable tutors to meet the learning needs of their adult learners so those learners can engage effectively with the texts, tasks and practices they encounter in their training and learning. The suggestions in each booklet are aligned with the following Tertiary Education Commission (TEC) publications:

- *Learning Progressions for Adult Literacy and Numeracy: Background Information*
- *Learning Progressions for Adult Literacy*
- *Learning Progressions for Adult Numeracy.*

These can be located on the TEC website at www.tec.govt.nz

These resources are based on research into effective adult literacy and numeracy, as described in *Lighting the Way*.[1] They also draw on school-sector work in literacy and numeracy, including Numeracy Project publications and the teachers' books *Effective Literacy Practice in Years 5 to 8* and *Effective Literacy Strategies in Years 9 to 13*.[2]

Readers are referred to the learning progressions publications (as listed above) to find more detailed discussions of adult learners, ESOL learners, and the theoretical basis for each of the progressions. The progressions books also contain glossaries and reference lists.

This set of resources has been developed to support the learning progressions. The suggestions are initial ideas only: they are aimed at helping tutors apply the learning progressions to existing course and learning materials. It is expected that tutors will use, adapt and extend these ideas to meet the needs of learners and their own teaching situations. There are many other resources available for tutors to use, and comparisons with the learning progressions will help you determine where other resources may fit in your programmes, and how well they might contribute to learner progress.

1 Ministry of Education (2005). *Lighting the Way*. Wellington: Ministry of Education.
2 Ministry of Education (2006). *Effective Literacy Practice in Years 5 to 8*. Wellington: Learning Media Limited.
Ministry of Education (2004). *Effective Literacy Strategies in Years 9 to 13*. Wellington: Learning Media Limited.

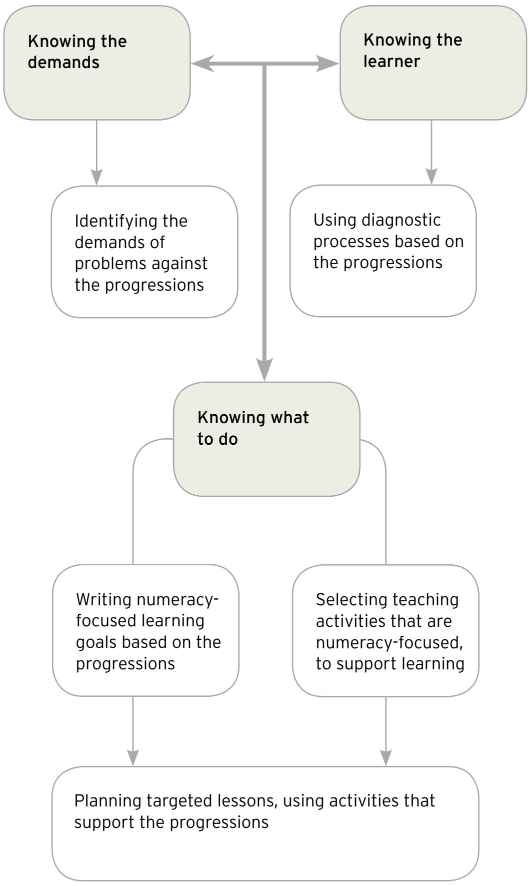
How to use this resource

There are three main sections in this resource:

- Knowing the demands (of the problems that learners want or need to manage).
- Knowing the learner (what they can do already, in order to determine the next learning steps).
- Knowing what to do (to help learners move on to the next steps).

These sections fit a process that can be illustrated as a flow chart.

Teaching adults to measure and interpret space: using the learning progressions



It is not essential to follow this order – in some circumstances, it will make sense to start by getting to know the learners before finding out what it is that they want to be able to do.

The following guide to working with this resource should be used alongside the information in *Learning Progressions for Adult Numeracy.*

Knowing the demands

First identify the numeracy demands of the course, task, text, problems or work practices the learners want to accomplish and map them against the learning progressions.

Knowing the learner

Use the tools in this section and the learning progressions to identify the learners' skills. This section contains two diagnostic activities that can be used to identify learners' strengths and needs in measurement.

Knowing what to do

Use the learning progressions to set achievable goals for and with the learners. The decisions about what to teach should be based on the identified demands of the tasks and where learners 'sit' on the learning progressions. Identify specific activities and materials to use (based on your course and context), then apply them in your teaching. Finally, review and reflect on the outcomes for the learners, with the learners.

In this resource, mapping the tasks the learners will encounter is the first step in planning for instruction. The next step is finding out where the learners 'sit' on the progressions. Where there is a gap between what the learners can do and what a task demands, you and your learners can refer to the learning progressions to make decisions about what to teach and learn next.

Strands and progressions

The learning progressions are organised within seven strands that cover the key components of listening, speaking, reading, writing and numeracy. Each progression shows a series of steps that reflect the typical sequence of skill development for oral language, written language and numeracy. The steps described are not tasks to be mastered in a set order. They do, however, offer information and a structure that can be used to develop curricula and learning and assessment tools. This current resource provides examples of how the progressions can be used. You are encouraged to design your own materials for teaching and learning to meet the needs of the adults with whom you work.

It is important to keep in mind that although the progressions are described in separate strands, in practice we use literacy, language and numeracy skills and knowledge in ways that are typically interconnected. For example, a person may **listen** to a report about rising interest rates, **speak** to their partner about their mortgage, **read** the information from several banks (using their knowledge of **numbers** to interpret and compare rates), then **write** questions to ask a bank about the options for managing a mortgage. Even filling in a form requires both reading and writing skills, and may also involve a discussion to clarify terms or requirements. Learners will better understand how their existing knowledge can support new learning when these connections are made clear.

Knowing the demands

Adult learners need to learn to measure and interpret space in order to solve particular kinds of problems for their individual purposes. Their tutors have to be able to analyse the problems their learners need to solve, and identify the demands and supports they present to learners. This section provides a guide to mapping (analysing) the kinds of problems adult learners need to be able to solve in relation to the learning progressions.

Mapping problems against the progressions

To determine the challenges of problems learners are expected to solve, you need to compare typical examples of these problems with the *Measure and Interpret Shape and Space* progressions and make decisions about where each problem fits with the relevant progressions. By comparing this information with what you know about the learners' knowledge and strategies skills, you will be able to determine the priorities for teaching and learning.

General process for all numeracy strands:

- Identify the strand or strands involved.
- Identify the progression or progressions involved.
- Identify the appropriate step in each applicable progression.

Example of a mapped problem

Office workplace requirements (from www.osh.dol.govt.nz)

For most office work today, space requirements are listed in terms of the average floor area per occupant. A value of 9 square metres is regarded as a minimum for an office worker who does not use a computer.

Mapping process

PROBLEM	SOLUTION	PROGRESSION(S) AND STEP(S)
What unit is best for measuring the width of a room? What unit is best for measuring the area of a room?	Metres for the width Square metres for the area	*Measurement*, 4th step
How many people without computers could fit into a rectangular office with wall lengths of 12 metres and 16 metres?	Area is 192 square metres 192 divided by 9 is 21.3 Estimate = a maximum of 21 people	*Measurement*, 5th step Multiplicative Strategies 4th or 5th step (depending on approach used)

Knowing the learner

General guidelines for using the diagnostic activities

- The aim of these diagnostic activities is to give a general picture about a learner's capability to understand and critically reflect on measurement problems they may be presented with in their daily lives.
- The diagnostic activities relate specifically to the *Measurement* progression. There are strong links, however, between all of the progressions and information from the diagnostic activity and questions may help you determine the next teaching and learning steps for a learner on all of the progressions.
- If the learner is finding the questions too difficult, it is not necessary to ask all questions.
- The diagnostic questions relate to authentic data. Ensure learners understand all language used.

At this stage there has been no development of diagnostic assessment items for the *Shapes and Transformations* progression and the *Location* progression. The reasons for this include:

- The skills and concepts contained in these progressions are less frequently taught in adult learning classes than the other progressions.
- The concepts of skills associated with these progressions are difficult to assess in a short time period. It is also believed that the *Measurement* progression will give sufficient information to the tutor to assess many of the learning needs of learners.

Measurement diagnostic activities

Two activities are presented to assess the learner's sense of measurement and measuring skills in relation to the *Measurement* progression.

The diagnostic questions for the Measurement progression start at the 4th step, as we believe the majority of learners will be at this step or at least able to attempt the questions at this step. If the learner is unsuccessful at the 4^{th} step, they should be rated at the previous step, which bridges the 2nd and 3rd steps.

Activity 1

The learners are asked to estimate and measure the length and weight of a book. The diagnostic questions increase in complexity as you move to the more advanced steps of the progression.

Resources needed

- A telephone directory or book (at least 2 centimetres thick).
- A ruler.
- Measuring scales.

	DIAGNOSTIC QUESTIONS FOR ACTIVITY 1	COMMENTS TO THE TUTOR
	1. How wide do you think this book is? 2. Using the ruler, measure the width of the book. 3. How many millimetres (or centimetres) would that be? 4. How heavy do you think this book is? 5. Using the scales, how heavy is the book? 6. How heavy is that in kilograms (or grams)?	Q1 and Q4 give an idea about whether the learner has a sense of the size of a centimetre (or millimetre) or weight of a kilogram. If the learner uses inches or pounds, ask if they can give the estimate in centimetres (or millimetres) and grams (or kilograms). Q2 and Q5 check if the learner can use measuring instruments for length and weight. Q3 and Q6 check if the learner can convert between centimetres and millimetres and between grams and kilograms.
	7. Where is the perimeter of the book? 8. The perimeter of this book is [ ] centimetres. What is that in metres? In kilometres? 9. What area does this book cover?	Q7 checks if the learner knows the term perimeter, but does not require them to measure and calculate it. Q8 checks if the learner can convert between centimetres and metres and kilometres. Q9 checks if the learner can calculate area. If they don't mention the measurement unit used, ask them to tell you the unit.
	10.What is the volume of the book?	Q10 checks if the learner can calculate the volume. Notice if they measure and multiply together the three dimensions or if they multiply the previously calculated area by the depth of the book.

Activity 2

The learners are asked to estimate and measure the dimensions of a glass or jar. The diagnostic questions increase in complexity at the more advanced steps of the progression.

Resources needed

- A straight-sided glass or jar.
- A measuring device marked in millilitres (ml).
- A ruler.
- A calculator.

	DIAGNOSTIC QUESTIONS FOR ACTIVITY 2	COMMENTS TO THE TUTOR
	1. How much water do you think this glass holds? 2. Use the measuring device to find the exact amount. 3. How many litres (or millilitres) is that?	Q1 gives information about whether the learner has a sense of the size of a millilitre. If the learner does not use metric units, ask if they can give an estimate in millilitres or litres. Q2 checks if the learner can use a measuring instrument for capacity. Q3 checks whether the learner can convert between millilitres and litres. If the answer to Q2 is given in litres, ask how many millilitres at Q3.
	4. The top of the glass is a circle. Measure its diameter. 5. How long is the radius? 6. Calculate the area of the circle. 7. Calculate the circumference (distance around the outside) of the circle.	Q4 Notice whether the learner measures across the widest part of the circle. If the learner does not know what the diameter of a circle is, do not proceed. Measurement diagnostic activity 1 gives information about understanding the concept of, and units of measurement used for, area and volume. Q5, Q6 and Q7 check whether the learner can explain how they are calculating these quantities.
	8. What is the volume of the glass?	Q8 gives information about whether the learner is able to calculate the volume of a cylinder. Notice whether the learner demonstrates understanding that the volume can be calculated by multiplying the area of the circle by the height.

Knowing what to do

General principles for guided teaching and learning activities

Each activity is aligned to a step on one of the *Measure and Interpret Shape and Space* progressions and is intended to strengthen the learners' understandings of the concepts and skills associated with that step.
The activities assume the learners understand the concepts and skills from earlier steps in the progression, so it is important that tutors check the learners have these understandings. Each teaching and learning activity includes the following components:

- A statement summarising the mathematical understandings that you will support the learners to develop in the activity.
- A list describing the teaching points covered in the activity.
- A list of resources or materials used in the activity. (These resources are readily available within adult learning settings and do not require the purchase of specialised materials.)
- A guided teaching and learning sequence that details the steps for you to use to develop the learners' knowledge of the skills and concepts addressed. The sequence includes questions for the tutors to pose to learners. The questions are intended to help the learners clarify their understandings as they explain their thinking to others. The questions are also designed to help tutors scaffold the learners' understanding of the concept or skill being explored.
- A follow-up activity for the learners to complete independently.

At this stage there has been no development of guided teaching and learning activities for the *Shapes and Transformations* progression and the *Location* progression.

Summary of guided teaching and learning activities

MEASUREMENT	
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Benchmarks for metres and centimetres

Measurement progression, 2nd–3rd steps

The purpose of the activity

In this activity the learners develop an understanding of the size of a metre and a centimetre by developing personal benchmarks for the two measures. The learners also develop their skill at estimating the length of objects in metres and centimetres.

The teaching points

- The learners will understand the need for having and using standard measures of length (metres and centimetres).
- External benchmarks (for example, visualising three 30-centimetre ruler lengths for 1 metre) can be used to conduct estimation tasks.
- The learners will be able to measure using rulers. (This includes the use of zero as the start point for taking a measurement or, if the ruler is 'broken', knowing the length measure is the difference between the start point and the end point.)
- Discuss with the learners the situations where it is important to know how long a metre and centimetre is.

Resources

- Metre length strips of paper or masking tape.
- Metre rulers.
- Centimetre cubes or centimetre squares.
- A selection of everyday objects under 10 centimetres in length.

The guided teaching and learning sequence

1. Begin the session by showing the learners a metre-long strip of paper or a metre length of masking tape stuck to the floor. Ask:

   "How long do think this strip of paper/tape is?" "Why do you think that?"

   Challenge the learners to think about how they came up with their estimate.

2. Show the learners a metre measuring stick. Ask:

   "Is the strip longer or shorter than a metre?"

3. Measure the strip of paper/tape using the metre stick/ruler to show that it is a metre. This is also an opportunity to ensure the learners appropriately use a metre ruler by lining up the start of the paper with the "0" mark on the ruler.
4. Next ask the learners to think about what they could do if they didn't have access to measuring tape or a ruler for measuring an object in metres. This discussion should focus on the importance of students developing personal benchmarks for measuring lengths. Your questioning may need to be more specific by posing prompts such as "How could you use parts of your body to show 1 metre in length?"
5. Give each learner or group of learners a metre length of paper or string. Ask them to use this to find a metre length on their body that could be used as a personal benchmark.
6. Once the learners have shared a number of possible personal benchmarks for 1 metre (for example, the length from their wrist to their shoulder, etc.) have them try to use it to find as many objects in the room that are 1 metre long (for example, the length of two desk tops, the height of a computer table). Have them record the objects they find that they estimate to be 1 metre in length.

7. Discuss with the learners the reason for having standard measures such as metres.

The rest of this guided learning sequence repeats the above steps with a centimetre length. This could be taken as a separate teaching session.

8. Show the learners a centimetre cube or a centimetre square of paper. Ask the learners:

   "How long do think a side of this cube or square is?" "Why do you think that?"

9. Ask individual learners or pairs of learners to find objects that are 2 centimetres, 5 centimetres, and 8 centimetres in length.

   "Why do you think that object is 2 centimetres? 5 centimetres? 8 centimetres long?"

   Challenge the learners to think about how they came up with their estimate.

10. Ask the learners to compare their lengths with another learner's or pair of learners' lengths and jointly decide which objects are the closest estimates to the three lengths.

11. Give the learners a centimetre cube or centimetre square of paper and say:

    "This is 1 centimetre in length. Do you think your estimate of the 5-centimetre-long object is too short, too long or about right?"

12. Get the learners to use the centimetre cube to measure the length of the 5-centimetre-long object. This is an opportunity to check that the learners are able to repeat the use of a single unit to measure length.

13. Ask the learners to think about what they could use as a personal benchmark for 1 centimetre. Your questioning may need to be more specific by posing prompts:

    "How could you use parts of your body to show 1 centimetre in length?"

14. Once the learners have shared a number of possible personal benchmarks for 1 centimetre (for example, the width of a fingernail, half the width of a thumb) have them use it to 'measure' a number of objects.

Follow-up activity

Ask the learners to cut strings of the following lengths using their personal benchmarks:

- 2 metres
- 9 centimetres
- 5 centimetres.

Understanding area

Measurement progression, 2nd–3rd steps

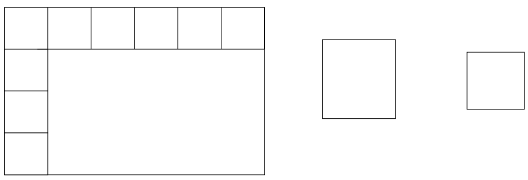
In this activity, the learners develop an understanding of area as a description of the number of square units needed to cover a shape. They do this by finding the area of rectangles, initially by counting square units, and then by imaging the number of square units.

The teaching points

- The area of a region is **the number of square units** needed to cover that region. (In this activity initially squares with sides other than 1 centimetre are used to encourage the learners to develop a conceptual understanding of area rather than use already known formulae.)
- For a rectangle, working out the number of squares in one row and one column and considering how many rows and columns are needed to cover the region is a faster way of calculating area than counting each square.
- Metric units for area are squares with sides of 1 centimetre and 1 metre and have the symbol cm^2 and m^2. (There are more: mm^2, hectare (10,000 m^2), km^2).

Resources

- Paper or cardboard rectangles (20 centimetres x 30 centimetres) and square units (5 centimetres x 5 centimetres and 2 centimetres x 2 centimetres) for each pair of learners.



- One square centimetre.
- Rulers including metre rulers or measuring tape.

The guided teaching and learning sequence

1. Give each learner a rectangle of paper (20 centimetres x 30 centimetres) and a square unit (5 centimetres x 5 centimetres) and ask them to find out how many squares cover the rectangle.
2. Ask the learners to share the results. Draw attention to the area of the paper being described by the number of square units needed to cover it – in this case 24 square units.
3. Discuss the method used. Some learners may move the square around and count the number, others may draw squares on the paper, others may count the number in one row and one column and multiply the two together.
4. Ask:

   "Was it necessary to draw all 24 square units to determine that it would take 24 units to cover the rectangular region?"

   Reinforce the idea that after finding the number of square units in one row and one column, the total number of squares can be obtained by considering how many rows and columns of squares will be needed to cover the region. (For example, if there are 4 squares in a row and 6 columns, there will be a total of 24 squares (6 x 4).)
5. Give each learner a square unit (2 centimetres x 2 centimetres) and ask them to find the area in terms of this square unit.
6. Ask the learners to share the results, emphasising the area as 150 square units, and the efficiency of counting squares in one row and column and multiplying the two together.

7. Ask the learners to discuss what units of measurement are commonly used for area. Listen for and reinforce 'square centimetre' and 'square metre'. (If acres are given, explain that this is not a metric measurement and that m^2 and hectares are the metric measurement used for land area.)
8. Show the learners a square with sides of 1 centimetre. Ask them how they could find the area of their piece of paper in square centimetres without using the centimetre square. You may need to prompt the use of a ruler to measure the length of the sides.
9. Ask them to find the area of the 20 centimetre x 30 centimetre paper in square centimetres, share the methods used, and record the result as 600 square units, 600 square centimetres and 600 cm^2. Draw attention to the unit and symbol.
10. Point out a large rectangle in the room (for example the door or a table) and ask the learners to estimate its area in square centimetres. Ask them to consider whether square centimetres are an appropriate unit to use to measure the rectangle's area and to suggest alternatives. Listen for 'square metres'.
11. Discuss with the learners what a square metre might look like. Ask each learner to make a square metre out of newspaper. Emphasise that a square metre is a square with sides of 1 metre with the symbol m^2, from 1 m x 1 m. Emphasise that it is the unit used to describe the area of larger shapes.
12. Ask the learners to estimate the area of a large rectangle in the room (floor, door or table) in square metres and share their estimations. Ask them to check their estimations using square metres. Discuss that fractions of the square metres may be necessary to cover the shape – "You would need 3 square metres, 2 whole square metres and 2 half square metres". Keep away, at this stage, from calculating the area to confirm estimates unless your learners are competent with multiplying fractions!

Follow-up activity

Ask the learners to choose small and large objects in the room, decide which metric unit of measurement they would use to describe the area, and give an estimation of the area of the object in that unit.

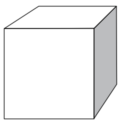
Understanding volume

Measurement progression, 2nd–3rd steps

In this activity, the learners develop an understanding of volume as a description of the number of cubic units needed to fill a solid shape. They do this by finding the volume of rectangular solids, initially by counting cubic units and then by forming a mental image of the number of cubic units that fit into the shape.

The teaching points

- Volume is the amount of space occupied by a solid expressed in cubic units.
- A cube has equal sides and equal square faces.



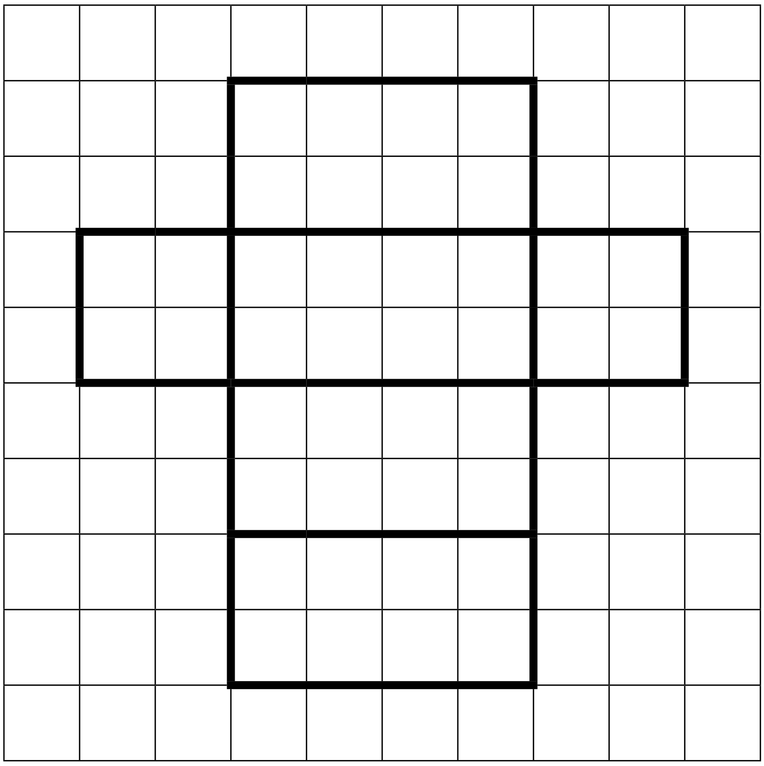
- For a rectangular solid, working out how many squares fit on one level and how many levels there are is a faster way of calculating the volume than counting each cube.
- Metric units for volume are cubic centimetres (cubes with sides of 1 centimetre) with the symbol of cm^3 and cubic metres (cubes with sides of 1 metre) with the symbol of m^3.
- m^3 means m x m x m and is said as metres cubed.
- Discuss with the learners the situations in which they might want to measure volumes.

Resources

- Nets with squares of 1 centimetre to build solid shapes (templates below).
- Lots of centimetre cubes.
- Rulers or measuring tapes that can measure 1 metre.
- Scissors.
- Sellotape.

The guided teaching and learning sequence

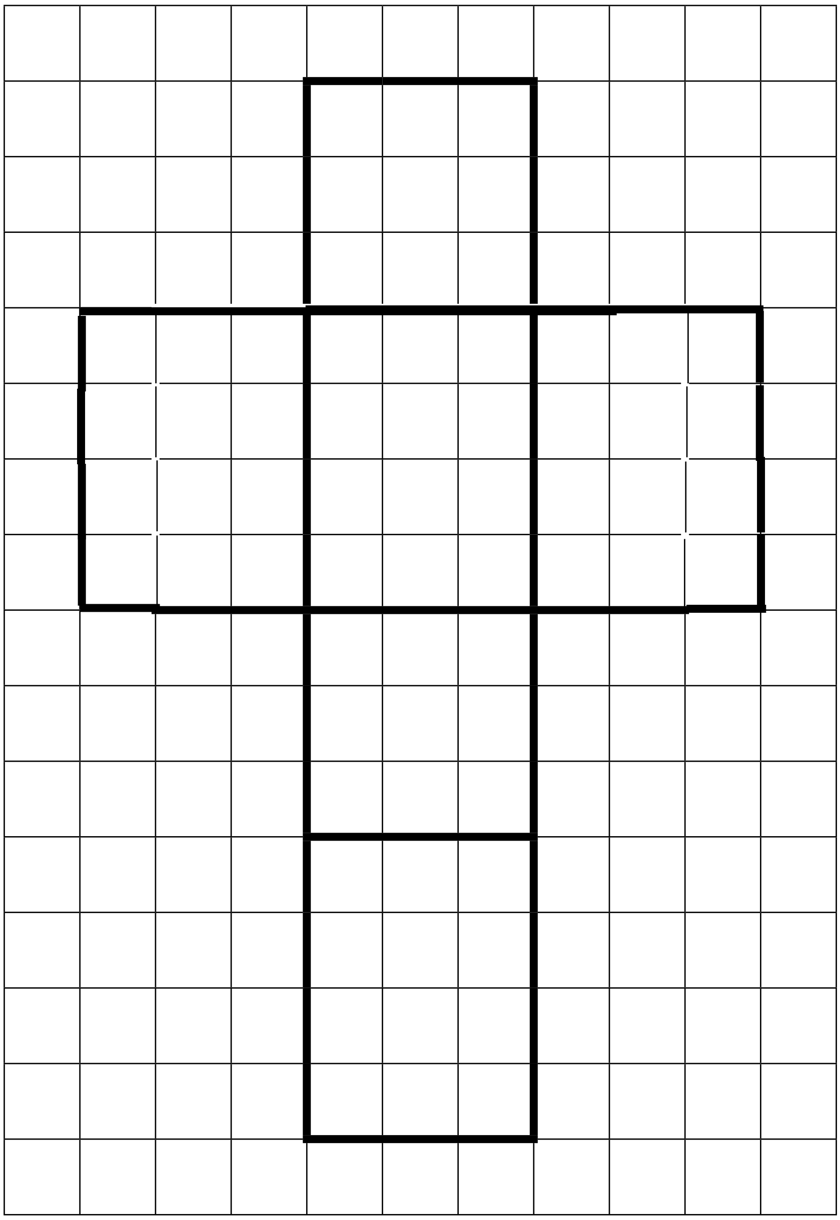
1. Ask the learners to build a solid from the net below (made from 1 centimetre squares) and to discuss how they might measure the space inside it. Listen for and encourage responses that suggest finding out how much fits in the solid.



2. Give each learner a 1 centimetre cube and ask them to describe its features. You may need to prompt them to measure the sides. Listen for and reinforce the fact that the length of the sides are all 1 centimetre and the faces are equal squares. Discuss the name "centimetre cube".
3. Ask the learners to predict how many cubes will fit into the solid. Record their predictions.
4. Ask them to check their predictions by filling the solid with cubic centimetres and share their results. Discuss the fact that the number of cubes that fill the space inside the solid is a way of measuring that space/volume and that, in this example, the volume is 16 cubes or 16 cubic centimetres or 16 cm^3.
5. Ask the learners to compare the results with their predictions. Encourage those with more accurate predictions to share their methods. Listen for and encourage, or prompt if necessary, comments like "8 cubes fit on the bottom, and they are stacked 2 high so there must be 16 cubes".

continued...

6. Ask the learners to make a solid from a second net, predict its volume and, if necessary, check the prediction by filling the shape with centimetre cubes (volume equals 36 cm^3).



7. Ask the learners to estimate the number of cubic centimetres that would fit into a freight container and whether cubic centimetres are an appropriate unit to measure its volume. Ask them to suggest an alternative unit. Listen for "cubic metres".

8. Discuss with the learners what a cubic metre might look like and how they might demonstrate it with newspaper. Using square metres of newspaper (as they made in the activity "Understanding area"), ask four learners to make a cubic metre. The learners will each hold a side face of a cubic metre. The bottom and top faces are contained within other faces. Emphasise that a cubic metre is a cube with sides of 1 metre with the symbol m^3, from 1 m x 1 m x 1 m. Emphasise that it is the unit used to describe the volume of larger shapes.
9. Ask the learners to estimate the volume of the room and other objects in the room (large cardboard box, carry bag, etc). You may need to discuss the use of fractions of a metre for objects with a volume smaller than 1 cubic metre.

Follow-up activity

Ask the learners to choose small and large solid objects in the room, decide which metric unit of measurement they would use to describe the volume and give an estimation of the volume of those objects.

Benchmarks for angle

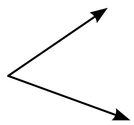
Measurement progression, 4th step

The purpose of the activity

In this activity, the learners develop an understanding of what angles are and how to measure them. The learners also develop their skill at measuring and estimating angles using degrees.

The teaching points

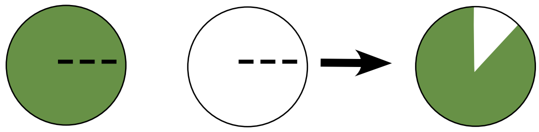
- Angles are composed of two rays that are infinite in length and meet at a common point (vertex).



- The attribute of angle size can be thought of as the *amount of turn to move from one ray to the other* or *the spread of the angle's rays*.
- The standard measure for measuring angles is degrees.
- Measuring using protractors. The protractor is generally poorly understood as a measuring instrument. Part of the difficulty occurs because the units (degrees) are very small; for example a single degree is physically impossible to cut out and use in much the same way that a single millimetre is too small to use. Another problem with protractors is that there are no visible angles showing on the protractor and only a series of marks around the outside. Finally, the numbering on most protractors is confusing with the numbers running both clockwise and anticlockwise. Ensure the learners understand the conventions for using protractors.
- Discuss with the learners relevant or authentic situations where the use of benchmarks for estimating angle is applicable.

Resources

- Paper circles of two different colours both cut along a radius and slid together along the cut to form an angle estimator. Turn one of the circles to create angles of different sizes with the contrast in the two colours creating the angle.



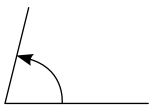
- Protractors.

The guided teaching and learning sequence

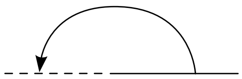
1. Begin by discussing rotations or turns in real situations. In these examples, focus the learners on the various ways of measuring the amount of turn about a particular point, starting at one position and finishing in another.

   Open a door and ask:

   "When you open a door to walk through it, how could you describe the amount that the door has turned?" (almost a $^1/_4$ turn)
   "Where is the point of the turn?" (hinge)
   "How could you draw this?"



2. Ask the learners to draw a half-turn and a full-turn.



3. Introduce the learners to the unit for measuring an amount of turn called the **degree**. Start by referring to a full turn as representing a turn of 360 units (or small amounts of turn) called degrees and denoted by "°".

4. Ask the learners to relate turns of one-quarter, one-half and three-quarters to a full turn of 360°. In particular, a one-half turn becomes 180°, a one-quarter turn becomes 90°, and a three-quarter turn becomes 270°.

   "What was the size of the angle when we opened the door?" (Almost a right angle or almost 90 degrees.)

5. Ask the learners to use the angle estimator to create the following angles:

   - 45 degrees
   - 200 degrees
   - 270 degrees.

   For each angle, ask the learners to check their estimates with other learners and explain how they made the estimate.

   "How did you estimate 45 degrees?"
   (half of a quarter turn or right angle)
   "How did you estimate 200 degrees?"
   (slightly more than half a turn)
   "How did you estimate 270 degrees?"
   (a three-quarter turn)

6. The angle estimator shows two amounts of turn. Ask:

   "How do you think they are related?"

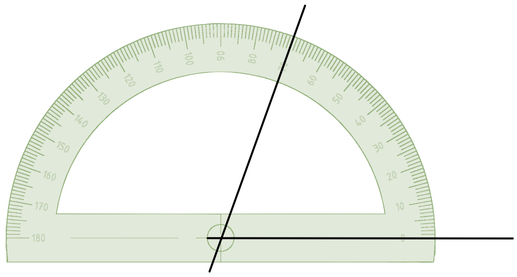
   The two angles together make up a full turn. If appropriate, you could introduce conjugate angles, which is the name given to two angles that combine to make a full-turn or 360 degrees. Supplementary angles is the name given to two angles that combine to make a half turn or 180 degrees.

7. Discuss with the learners the usefulness of using 'benchmark turns' for estimating angles (for example, using a quarter turn (right angle or 90 degrees) as a benchmark). Relate the need for angle measurements to the workplace or course as appropriate.

8. Give the learners protractors, explaining that they are the measuring tool used to measure angles. With the learners working in pairs, ask them to discuss how the protractor works. Encourage them to reflect on the angle estimators.

9. Ask the learners to use protractors to measure accurately one-quarter, one-half and three-quarter turns. Let them also measure accurately 30°, 60° and 120°.

10. Ask the learners to describe how they would help someone who had incorrectly measured the following angle as 110°.



Follow-up activity

Ask pairs of learners to take turns drawing angles. They give the angle to their partner who first draws a rough estimation of the angle and then, using the scale on the protractor, draws the accurate value.

Benchmarks for weight

Measurement progression, 4th step

The purpose of the activity

In this activity, the learners develop an understanding of weight and mass. The learners also develop their skill at using benchmarks of 1 kilogram and 500 grams to aid in estimating the weight of given objects.

The teaching points

- The learners will understand the need for having and using standard measures of weight (kilograms and grams).
- The learners will know the conversions between kilograms and grams (1,000 grams = 1 kilogram)
- The learners will develop and then use benchmarks (for example, visualising the weight of 1 kilogram) to carry out estimation tasks.
- Discuss with the learners relevant or authentic situations where the understanding of kilograms and grams is necessary.

Resources

- Weighing devices for objects less than 3 kilograms (as applicable: electronic, analogue, spring, balance scales).
- Objects that weigh 1 kilogram (for example, 1 litre of milk, a 1-kilogram block of cheese) or $^1/_2$ kilogram (for example, a block of butter, a tub of margarine).
- A range of objects.
- Supermarket bags.

The guided teaching and learning sequence

1. Begin by showing the learners three 'closed' supermarket bags (labeled a, b, c) and posing the questions:

   "Which is the heaviest?"
   "Which is the lightest?"

   Give the learners the opportunity to lift the bags and make their decision.

   Check that everyone agrees on the heaviest and lightest bags. If not, ask those who disagreed or who were uncertain to directly compare two of the bags by holding one in each hand.

   Ask the learners if they have any idea about the weight of the heaviest bag.

   "How heavy do you think the heaviest bag is?"
   "Why do you think that?"

2. Discuss the difficulty of estimating the weight of objects. This is usually due to the fact that we don't have much practice at estimating weight. Also we don't have the same 'personal' means with which to benchmark like we do for length (for example, fingertip to shoulder is about 1 metre).

3. Ask the learners to suggest objects that could be used as benchmarks for a 1-kilogram weight. Possibilities include: 2 tubs of margarine, 2 blocks of butter, a litre of milk or juice, a kilogram of cheese.

4. Check the learners understand that the metric prefix 'kilo' means 1,000.

   "What does kilo stand for in kilogram?"
   "How many kilograms in 2,000 grams?"
   (2 kilograms)
   "How many grams in 6 kilograms?"
   (6,000 grams)

5. Distribute a 1 kilogram and 500 gram object to the learners working in groups. Give the learners the opportunity to handle the objects.
6. Prepare a number of labelled bags filled with objects that in total weigh between 200 grams and 3 kilograms. Ask the learners to estimate the weight of each bag using the following ranges:
   - under 500 grams
   - 500 grams to 1 kilogram
   - 1–1.5 kilograms
   - 1.5–2 kilograms.
7. Examine with the learners the weighing devices that you have available, discussing the level of accuracy permitted by each device. Also discuss with the learners the purpose of having standard units of measure so weights can be accurately communicated.
8. Ask for volunteers to select one of the bags from above and weigh this on a set of scales. Use this to check the accuracy of estimated weights and to reinforce the correct use of the weighing device.
9. Ask:

   "What would you use to estimate 20 grams if you didn't have a measuring device?"
10. Next ask learners to work in pairs or groups to fill a plastic bag with rice so that it weighs 20 grams.
11. Give the learners the opportunity to check their estimates on the measuring devices.

Follow-up activity

Ask the learners to fill a bag with objects until they estimate that the bag weighs 1 kilogram. Use measuring devices to find the 'winner'.

Benchmarks for capacity

Measurement progression, 4th step

The purpose of the activity

In this activity, the learners develop an understanding of the size of 1 litre and $^1/_2$ litre. They learn to express these amounts in millilitres. They develop their skill at estimating the volume of containers in litres and millilitres by establishing personal benchmarks.

The teaching points

- The learners will develop an understanding of the size of 1 litre and $^1/_2$ litre.
- 1 litre = 1,000 millilitres and $^1/_2$ litre = 500 millilitres.
- The symbols for litre are l or L, and the symbols for millilitre are ml or mL.

Note: The metric system symbols do not use 's' for the plural: 50 millilitres is written as 50 ml or mL not mls or mLs.

- The learners will understand the use of benchmarks (for example, a water bottle and a beer can where the volume is known and remembered) to estimate volumes in litres and millilitres.
- The learners will be able to read scales for measuring volume.
- Discuss with the learners the fact that the terms volume and capacity are both used to describe the amount of space a solid occupies. They are sometimes distinguished, with capacity being used for how much a container can hold (measured in litres or its derived units) and volume being how much space an object displaces (measured in cubic metres or its derived units.)

Resources

- Each learner to find and bring one empty litre container, for example water bottle or milk container.
- Containers (smaller than 1 litre) with the volume written on them, such as soft drink or beer cans.
- Containers larger than 1 litre such as a bucket, electric jug.
- A variety of empty containers, some smaller and some larger than 1 litre, without the volume written on them, such as jars and glasses.
- Sand or water for measuring volume.
- Measuring cup, jug and/or cylinder marked in millilitres.

The guided teaching and learning sequence

1. Ask the learners to think about situations where they might need to have an understanding of the size of litres and millilitres. Possible examples are judging whether you have sufficient oil for an oil change, whether the amount of liquid in a holding tank is small enough to be able to be released into the drain, catering, giving medicines, etc.
2. Give each group of learners five or six containers and ask them to decide which are smaller and which are larger than 1 litre. Encourage them to use the litre container they have brought to make the decisions.
3. If the learners are unsure whether a container holds more or less than 1 litre, ask them to find out by filling the container with sand or water and tipping the contents into a known litre container – be prepared for spills!

4. Discuss using their litre container as a benchmark for estimating volumes in the future.

   "Estimate how many litres in a standard bucket, in an electric jug, in a sink."

5. Explore the connection between litres and millilitres by asking:

   "What does milli mean?" (one thousandth ($^1/_{1000}$) part of)
   "How many millilitres in 1 litre, 2 litres, $^1/_2$ litre, $^1/_4$ litre?" (1,000 mL, 2,000 mL, 500 mL, 250 mL)
   "How many litres in 3,000 mL, 1,500 mL?" (3 L, $1^1/_2$ L)

   Using containers that are smaller than 1 litre, ask the learners to decide which are smaller and which are larger than $^1/_2$ litre/500 mL. Get them to check their decisions by filling the container with sand or water and using the measuring cup, jug and/or cylinder marked in millilitres.

6. Ask the learners to try to find out the volume. Be familiar with the scale on the device so you can show the learners how to read an accurate measurement.

7. Share all results with the whole group so the learners see and are told the volume of a large range of containers.

8. Show the learners containers for which the volume is given and read out the volume. Demonstrate how the learners could use one of these as a benchmark for estimating millilitre volumes. For example, if you know the beer can holds 330 mL, estimate the volume of the wine bottle.

9. Using their litre and millilitre benchmarks, ask the learners to estimate the volume of containers not already measured. Possibilities include coffee cups, glasses, a wine bottle, a large juice container, a medicine cup, a teaspoon. Ask the learners to work in groups and share and justify their decisions.

Follow-up activity

Ask the learners to estimate and read/measure the volume of containers before the next session and to bring one to share with the group at the next session.

Time formats

Measurement progression, 4th step

In this activity, the learners become familiar with expressing time in each of its three forms: analogue, 12 hour, and 24-hour digital.

The teaching points

- There are 60 minutes in one hour.
- There are three ways of expressing time:

ANALOGUE	DIGITAL	24-HOUR
11 o'clock	11:00 am/pm	11:00/23:00
5 past 11	11:05 am/pm	11:05/23:05
10 past 11	11:10 am/pm	11:10/23:10
quarter past 11	11:15 am/pm	11:15/23:15
20 past 11	11:20 am/pm	11:20/23:20
25 past 11	11:25 am/pm	11:25/23:25
half past 11	11:30 am/pm	11:30/23:30
25 to 12	11:35 am/pm	11:35/23:35

- On an analogue clock, there is a hand to indicate the hours and another to indicate the minutes. One circuit of the minute hand is 60 minutes or one hour. Each division is 5 minutes. Time is expressed as minutes past the hour until 30 minutes past and then is usually expressed as minutes to the hour. 15 minutes past the hour is quarter of the circuit and is often called quarter past the hour. 30 minutes past the hour is half the circuit and is often called half past the hour. 15 minutes to the hour is also called quarter to the hour. There is nothing to indicate whether it is morning or afternoon.
- On digital clocks, time is expressed as hours: minutes. The minutes are always expressed as past the hour, counting on from :00 until :59. There are still 60 minutes in the hour so :00 is o'clock and :30 is still half past the hour.
- 24-hour time is expressed as hours:minutes (or without a gap). The hours are expressed from 00 to 23 (00:00 being midnight). There is no confusion between morning or afternoon.
- Discuss with the learners the fact that morning or afternoon is usually indicated by "am" (from the Latin *ante meridiem/meridian* – before noon) or "pm" (*post meridiem/meridian* – after noon).

Resources

- A selection of analogue clocks and watches and digital clocks and watches, some of which show 24-hour time.

The guided teaching and learning sequence

1. Begin by brainstorming the words used to talk about time and recording these on the board. For example: hours, minutes, seconds, 10 to or past, quarter to or past, half past, o'clock, two fifty.
2. Explain that there are three different ways in which clocks show time and ask the learners if they can describe them.
3. Show the learners an analogue clock and ask the following questions. Record key points on the board.

   "How much time is one complete circuit of the clock?"
   "Give this time in two different units"
   (1 hour, 60 minutes)
   "How much time in one segment of the clock?"
   (5 minutes)

4. Set the clock to 10 past 1 and ask the following questions:

   "What is the time?"
   "Are there other ways in which this can be said?" (10 past 1, one ten)
   "How do you know it is 10 minutes past?" (two segments of 5 minutes each)

5. Set the clock to half past 2 and ask:

   "What is the time?" "Are there other ways in which this can be said?" (half past 2, two thirty)
   "How do you know?" (half way round the clock, whole clock 60 minutes, half is 30, six segments of 5 minutes)

6. Ask for a volunteer to set the clock to other times when fractions are used (quarter past, quarter to) and to explain why they are used (a complete circuit of the clock represents one hour, at quarter past the hour one quarter of that circuit has been completed, at quarter to the hour one quarter of the circuit remains to go).

7. Divide the learners into groups and give each group two digital clocks, one with and one without 24-hour time. Ask the learners to identify similarities to and differences from an analogue clock.

   Listen for and reinforce one hour is 60 minutes, the first two digits express hours, the second two minutes, time is always reported in minutes past the hour, the hours run from 00 to 12 or 00 to 23 (the latter indicating the difference between morning and afternoon), midnight is 00.

Follow-up activity

Divide the learners into groups of three with each learner in the group taking on the role of analogue time, digital time or 24-hour time. One names a time, and the other two give it in their format. Encourage the learners to change around to practise all forms of time.

When they are familiar with the formats, get them to ask each other questions, such as:

"What is the time in half an hour?"
"What will the time be in 11 hours?"
"What was the time 2 hours ago?"
"What will be the time in 35 minutes?"
and to give the answer in all formats.

Time conversions

Measurement progression, 4th step

In this activity, the learners become familiar with converting from one unit of time to another.

The teaching points

- Some 'conversion factors' for time are 60 minutes in one hour, 24 hours in one day and seven days in a week.
- When you change from one unit of time to another unit of time, the amount of time stays the same; you are just giving it a different name.
- If you are converting from a smaller unit of time to a bigger unit of time, you expect less of the bigger unit and therefore divide by the conversion factor. For example, to change minutes (small) to hours (big) you divide by 60.
- If you are converting from a bigger unit of time to a smaller unit of time, you expect more of the smaller unit and therefore multiply by the conversion factor. For example, to change weeks (big) to days (small) you multiply by seven.

  Note: These last three principles apply whenever you want to change a quantity given in one unit of measurement into another unit of measurement, for example, changing a measurement made in millimetres into metres.
- Developing conceptual understanding of division. One of the reasons learners have difficulty solving word-based problems is that they do not have sufficient conceptual understanding of addition, subtraction, multiplication, and division (particularly division) to make correct choices of which to use to solve a problem.
- Discuss with the learners the fact that time is not a decimal system like our number system. When problems with time are solved using a calculator and the answer includes a decimal fraction, this represents tenths, hundredths or thousandths of a unit of time rather than days, minutes, etc. For example, 2.4 hours is 2 and 4 tenths hours (2 hours 24 minutes) not 2 hours 4 minutes.

Resources

- Calculators.

The guided teaching and learning sequence

1. Write "120 minutes" on the board and ask the learners how many hours it is. When they reply "two hours", ask "How did you work that out?"

   In the discussion, listen for and highlight the fact that you have to know that one hour is 60 minutes and you have to find out how many 60 minutes are in 120 minutes.

   Write "How many 60 minutes are in 120 minutes?" on the board and highlight the use of division by writing 120 divided by 60, $120 \div 60$, and 120/60 next to the question.
2. Write "21 days" on the board, and ask the learners how many weeks it is.
   Repeat the process from 1 above.
3. Write "five hours" on the board and ask the learners how many minutes are in five hours. When they reply "300 minutes", ask "How did you work that out?"

   In the discussion, listen for and highlight the fact that you have to know that one hour is 60 minutes, and you have five hours, so that is five times 60 minutes or 300 minutes.

   Write the words and the symbols for five times 60 minutes on the board.

4. Write "six weeks" on the board and ask the learners how many days are in six weeks. Repeat the process from 3 above.
   (If the learners are able to multiply with multi-digit numbers, you could ask how many hours are in six weeks).
5. Ask:

   "How do you know when to multiply and when to divide?"

   Listen for, and reinforce, the response that you multiply when you change from a bigger unit to a smaller unit, and divide when you are moving from a smaller unit to a bigger unit.
6. Draw attention to the fact that 120 minutes is two hours and ask:

   "Is the amount of time the same in both cases?"

   Repeat with six weeks is 42 days. Emphasise that although the units are different, the amount of time remains unchanged.
7. Write "144 minutes" on the board and ask the learners how many hours and minutes it is. Ask the learners to share strategies.

   Some learners may have used a calculator, or you may wish to ask them to use a calculator if they are likely to in daily life. Using a calculator, 144 divided by 60 will give 2.4.

   Ask:

   "2.4 what?"
   "If I told you to be back in 2.4 hours, would you know what time to be back?"
   "What does 0.4 hours mean?" (four-tenths of an hour)
   "What is one tenth of an hour?" (6 minutes)
   "What is four-tenths of an hour?" (24 minutes)
   "What is 2.4 hours in minutes?" (144 minutes)

Follow-up activity

Prepare some questions for the learners to work on in groups, for example:

- Each learner in the room is going to make a 15-minute presentation to the class, how many hours and minutes will it take for the whole class to present?
- We have two periods of 1 hour 50 minutes for presentations and 10 learners, how long will each learner get?
- A person is to have 240 milligrams of a drug per day in three doses evenly spread throughout the day. How long between each dose and how much is each dose (Extension: If the first dose is at 14:00, what time are the other two doses)?
- I want to set my video recorder to record two programmes, three-quarters of an hour long and the other 1 hour 10 minutes long. There is a 12-minute break in the middle. If I set the recording to start at 20:00, give a time to set it to stop so that both programmes will be recorded.

Connecting kilometres, metres, centimetres and millimetres

Measurement progression, 5th step

The purpose of the activity

In this activity, the learners develop understanding of the relative size of metres, centimetres and millimetres by measuring the length of objects, using a variety of measuring tools and scales.

The teaching points

- The learners will be able to determine the appropriate standard unit for measuring a given length.
- The learners will use knowledge of place value to understand the conversions between millimetres, centimetres and metres.
- Discuss with the learners relevant or authentic situations where the conversion between units of length is applicable.
- Ensure the learners understand that the unit needs to be included with the length measure so the size or value of the measure is not misunderstood.
- Use personal benchmarks to estimate the length of objects.

Resources

- A variety of rulers or measuring tapes marked in millimetres, centimetres and metres. Include some 'broken' rulers that do not have a zero starting point.
- Conversion dominoes for the follow-up activity (Appendix A).

The guided teaching and learning sequence

1. Give the learners a piece of paper with an unlabelled 25-centimetre line drawn on it and ask the learners to estimate its length.
2. Ask the learners to share their estimates. As they give their estimates, ask the learners to explain what they used to make those estimates.

   "Why did you think it was that length?"
   "What do you use to guide your length estimates? in centimetres? in metres? in millimetres?"

   Their answers will allow you to see if the learners have established personal benchmarks for centimetres, millimetres or metres.
3. Next ask the learners to use a ruler or measuring tape to measure the length of the line and record this beside the line.
4. Ask the learners to work in groups to compare their recorded lengths.

   "Is your recorded length the same as everyone else's? If not, why are they different?"
   "Did you all use the same measurement unit?"
   "How can you use a 'broken' ruler to measure an object?"
5. Ask the groups to share their records with the class. Record the lengths on the board. If the group has not given a unit, then do not record it with the length.
6. Discuss any differences in the recorded lengths.

   "Is 25 the same as 25 centimetres?"
   "Is 250 millimetres the same as 25?"

   Encourage the learners to realise that the unit must be included or the length may be misunderstood.

7. Write "37 centimetres" on the board and ask volunteers to draw a line that length on the board without using a ruler or measuring tape. Ask the learners to explain what they are using to make their estimate of 37 centimetres.

   "What do you use to estimate 37 centimetres?"
   "How accurate do you think you are?"

8. If they have no reference point, suggest that they think of a 30-centimetre ruler and add about another quarter of a ruler length. Alternatively, they may know the length of their hand span (about 20 centimetres) and use slightly less than two hand spans to estimate 37 centimetres.

9. Ask the learners how else the measure 37 centimetres could be recorded.

   "How else can we correctly record the length 37 centimetres?"

   Record the responses on the board. Encourage the learners to explain why their answer is correct.

   "Why is 370 millimetres the same as 37 centimetres?"
   "Why can this be also written as .37 metres?"

10. Draw a place-value chart on the board as follows:

TENS OF THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES	TENTHS	HUNDREDTHS
			3	7 •		

   "Is 37 correctly recorded?"

   The answer "yes" to this requires the learners to identify that the "unit" for the ones column is centimetres.

continued...

11. Record centimetres with the 'ones' and then ask the learners to identify where millimetres, metres, and kilometres are in relation to the centimetres. Discuss the placement of the decimal point between ones and tenths.

Centimetres

HUNDREDS OF THOUSANDS OF CENTIMETRES (KILOMETRES)	TENS OF THOUSANDS OF CENTIMETRES	THOUSANDS OF CENTIMETRES	HUNDREDS OF CENTIMETRES (METRES)	TENS	ONES (CENTIMETRES)	TENTHS OF A CENTIMETRE (MILLIMETRES)	HUNDREDTHS OF A CENTIMETRE
				3	7 •		
			3	5	0 •		
					4 •	5	
			5	0	0 •		

Ask the learners to record the following measurements on a copy of the chart above:

- 3.5 metres (350 centimetres)
- 45 millimetres (4.5 centimetres)
- 500 centimetres.

12. Record metres with the 'ones' and then ask the learners to identify where millimetres, centimetres and kilometres are in relation to the metre. Note this requires the addition of an extra column (thousandths of metres or millimetres).

Metres

TENS OF THOUSANDS OF METRES	THOUSANDS OF METRES (KILOMETRES)	HUNDREDS OF METRES	TENS OF METRES	ONES (METRES)	TENTHS OF A METRE	HUNDREDTHS OF A METRE (CENTIMETRES)	THOUSANDTHS OF A METRE (MILLIMETRES)
				•	0	3	7
				•	5	6	
				•	0	4	5
				5 •			
4	5	0	0	0 •			

Ask where 3.7 centimetres would be recorded on this chart. Discuss why this is recorded as 0.037 metres.

13. Ask the learners to record the following measurements on the chart and then write the measure in relation to metres:
    - 56 centimetres (0.56 metres)
    - 45 millimetres (0.045 metres)
    - 500 centimetres (5 metres)
    - 45 kilometres (45,000 metres).
14. Discuss how measurements can be converted without the use of a chart.

    "How do you convert metres to centimetres without using the chart?" Or: "Why do you multiply the number of metres by 100 to convert to centimetres?"
    "How do you convert metres to kilometres?" Or: "Why do you divide the number of metres by 1,000 to convert to kilometres?"
    "How do you convert millimetres to kilometres?" Or: "Why do you divide the number of millimetres by 1,000,000 (million) to convert to kilometres?"

Follow-up activity

Conversion dominoes played by groups of two to four learners.

1. The dominoes are placed face down on the table and mixed well.
2. Each player takes up dominoes to make up their hand. The number of dominoes taken up depends on the number of players: two players draw seven dominoes each, three players draw five dominoes each, and four players draw five dominoes each. The remainder of the dominoes are held in reserve.
3. The player with the double domino that has the 'longest' length recorded on it places the first domino. Play proceeds in a clockwise direction. Each player adds a domino to an open end of the layout, if possible.
4. If a player is unable to make a move, they take up a domino from the reserve and the turn passes to the next player. If there are no dominoes left for the player to take up, then the player must pass.
5. A game ends either when a player plays all their tiles or when a game is blocked. A game is blocked when no player is able to add another tile to the layout.

Connecting tonnes, kilograms and grams

Measurement progression, 5th step

The purpose of the activity

In this activity, the learners develop an understanding of the relative size of tonnes, kilograms and grams and the conversions between the units.

The teaching points

- The learners will be able to determine the appropriate standard unit for a given measuring (weight) context.
- The learners will use knowledge of place value to understand the conversions between tonnes, kilograms and grams.
- The learners will know that measuring weight to appropriate levels of precision depends on the task and the scale being used.
- The learners will understand that mass and weight are not the same thing. Mass is the amount of matter in an object and a measure of the force needed to accelerate it. Weight is a measure of the pull or force of gravity on an object. If the same force is applied to several objects, those with a smaller mass will be affected the most. The weight of an object is its mass under the force of gravity. Because gravity is close to constant on Earth, we judge an object's mass by weighing it. If an object were taken to the moon, its mass would be the same as on Earth, but its weight would be considerably less because the gravity on the moon is far weaker than on Earth. For practical purposes, the measures of mass and weight are about the same, and the terms are used interchangeably.
- The learners will understand that mass is not proportional to volume. For example, a piece of lead has a far greater mass than a similar-sized piece of foam plastic.
- Discuss with the learners relevant or authentic situations where the conversion between units of weight is applicable.

Resources

- A range of measuring devices.
- Several 'filled' bags that weigh 1.4 kilograms.
- Conversion dominoes for the follow-up activity (Appendix B).

The guided teaching and learning sequence

1. Give the learners an unlabelled bag that weighs 1.4 kilograms and ask them to estimate its weight.
2. Ask the learners to share their estimates. As they give their estimates, write the estimates on the board and ask the learners to explain what they used to make their estimate.

   "Why did you think it was that weight?"
   "What do you use to guide your weight estimates – in grams? in kilograms? in tonnes?"
3. Their answers will allow you to see if the learners have established personal benchmarks for units of weight.
4. Next ask the learners to use one of the weighing devices to weigh the bag. Ask the learners to work in groups of three or four to compare their recorded weight with the weights from others in the group.

   "Is your recorded weight the same as everyone else's? If not, why are they different?"
   "Did you all use the same measurement unit?"
5. Ask the groups to share their records with the class. Record the weights given on the board. If the group has not given a unit, then do not record it with the weight.
6. Discuss any differences in the recorded weights.

   "Is 1.4 the same as 1.4 kilograms?"
   "Is 1,400 grams the same as 1.4 kilograms or 1.4?"

   Encourage the learners to realise that the unit must be included or the weight may be misunderstood.

7. Draw a place-value chart on the board as follows:

THOUSANDS	HUNDREDS	TENS	ONES		TENTHS	HUNDREDTHS	THOUSANDTHS
			1	•	4		

"Is 1.4 kilograms correctly recorded?"

The answer "yes" to this requires that the learners identify that the 'unit' for the ones column is kilograms.

8. Record kilograms with the 'ones' and then ask the learners to identify where grams and tonnes are in relation to the kilograms. Discuss the placement of the decimal point between ones and tenths.

Kilograms

THOUSANDS OF KILOGRAMS (TONNES)	HUNDREDS OF KILOGRAMS	TENS	ONES (KILOGRAMS)		TENTHS OF A KILOGRAM	HUNDREDTHS OF A KILOGRAM	THOUSANDTHS OF A KILOGRAM (GRAMS)
			1	•	4		
			0	•	6		
5	5	6	0	•			
			3	•	0	5	5

Using this chart, ask the learners to record the following measurements on it:

- 600 grams (0.6 kilograms)
- 5,560 kilograms (5.56 tonnes)
- 3,055 grams (3.055 kilograms).

9. Record grams with the 'ones' and then ask the learners to identify where kilograms and tonnes are in relation to the grams. Note that this requires the addition and relabelling of columns.

continued...

Grams

MILLIONS OF GRAMS (TONNES)	HUNDREDS OF THOUSANDS OF GRAMS	TENS OF THOUSANDS OF GRAMS	THOUSANDS OF GRAMS (KILOGRAMS)	HUNDREDS OF GRAMS	TENS OF GRAMS	ONES (GRAMS)	TENTHS OF A GRAM
			3	5	0	0 •	
2	5	0	0	0	0	0 •	
						•	

Ask how 3,500 grams would be written on this chart. Discuss how else this could be recorded.

"How else could you record 3,500 grams?" (3.5 kilograms)
"How do you write 2.5 tonnes on this chart?"
"How else could you record 2.5 tonnes?" (2,500,000 grams, 2.5 million grams, 2,500 kilograms)

10. Give each of the learners, or pairs of learners, a grams chart and ask them to record the following weights on the chart and then convert to kilograms and tonnes. Discuss their answers.
    - 34,500 grams (34.5 kilograms, 0.0345 tonnes).
    - 124,589 grams (124.589 kilograms, 0.124589 tonnes).

11. Ask the learners to record the following weights on their grams chart and then convert to grams and kilograms.
    - 6.7 tonnes (6,700,000 grams, 6,700 kilograms).
    - 30 tonnes (30 million grams, 30,000 kilograms).
    - 0.05 tonnes (50 kilograms, 50,000 grams).

12. Discuss how measurements can be converted without using a chart.

    "How do you convert kilograms to grams without using the chart?" Or: "Why do you multiply the number of kilograms by 1000 to convert to grams?"
    "How do you convert grams to kilograms?" Or: "Why do you divide the number of grams by 1,000 to convert to kilograms?"
    "How do you convert grams to tonnes?" Or: "Why do you divide the number of grams by 1,000,000 (one million) to convert to tonnes?"

13. Discuss with the learners the difference between mass and weight using their existing understandings as a starting point for the discussion.

    "Sometimes the weight of an object is called its mass. Are they the same thing?"

Follow-up activities

- Ask the learners to give their weight in kilograms, grams and tonnes.
- Have the learners play conversion dominoes* as groups of two to four learners.
    1. The dominoes are placed face down on the table and mixed well.
    2. Each player draws dominoes to make up their hand. The number of dominoes drawn depends on the number of players: two players draw seven dominoes each, three players draw five dominoes each, and four players draw five dominoes each. The remainder of the dominoes are held in reserve.
    3. The player with the double domino with the 'largest' weight recorded on it places the first domino. Play proceeds clockwise. Each player adds a domino to an open end of the layout, if possible.
    4. If a player is unable to make a move, they draw a domino from the reserve and the turn passes to the next player. If there are no dominoes left, then the player must pass.
    5. A game ends either when a player plays all their dominoes or when a game is blocked. A game is blocked when no player is able to add another domino to the layout.

* Conversion dominoes activity courtesy of BMET tutor, Emily Harrop.

Fixed area rectangles and perimeters

Measurement progression, 5th step

The purpose of the activity

In this activity, the learners develop an understanding of how to calculate the area and perimeter of rectangles from the measurement of side lengths. They learn that the perimeters of rectangles of fixed area do not remain the same.

The teaching points

- The learners understand that the perimeter of a rectangle is calculated by adding together the four side lengths or by adding together two adjacent side lengths and doubling.
- The learners understand that the area of a rectangle is calculated by multiplying together the lengths of two adjacent sides.
- The learners understand that the rectangle with the shortest perimeter for a fixed area is a square.
- The learners measure length using centimetres and millimetres, and area using square centimetres (cm^2) and square millimetres (mm^2).
- Discuss with the learners relevant or authentic situations where measuring the perimeter or area of rectangles is applicable.

Resources

- Rulers and measuring tapes (marked in centimetres, millimetres).
- 1 centimetre grid paper (Material Master).
- Scissors.
- A selection of rectangular and square objects.

The guided teaching and learning sequence

1. Give each of the learners the grid paper and ask them to cut out a 4 centimetre x 4 centimetre square. Ask:

   "What is the area of the square?"
   "How do you record that?" (16 cm^2)
   "How did you calculate the area?"

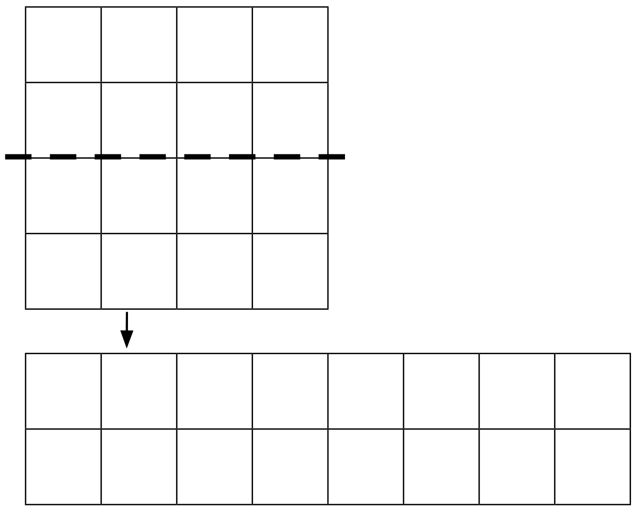
   If the learners counted squares, ask if they could do it another way. Encourage them to notice that you multiply the side lengths together, which in the case of a square is the same as squaring the side length: $4 \times 4 = 4^2 = 16$).

2. Next ask:

   "What is the perimeter of the square?"
   (Check the learners all understand that the perimeter is the distance around the sides of the square)
   "How do you record that?" (16 cm)
   "How did you calculate the perimeter?"

   If the learners added the four side lengths, encourage them to notice that you can calculate the perimeter by multiplying one of the side lengths by four (as the four sides of a square are of equal length).

3. Ask the learners to cut the square into two pieces with a single straight scissor cut, so that the two pieces when moved and rearranged form a rectangle with a perimeter of 20 centimetres.



Ask for a volunteer to demonstrate how they formed a rectangle with a 20 centimetre perimeter from the square.

4. Ask:

   "What is the area of this rectangle?"
   "How did you know or work that out?"

   Encourage the learners to notice that the area is unchanged.

5. Ask:

   "Can you make any other rectangles that have an area of 16 cm^2?" (16 x 1 rectangle)
   "What is the perimeter of this rectangle?"
   (34 centimetres)

6. Next ask the learners to cut a 12 centimetre x 3 centimetre rectangle from squared paper and record its area and perimeter on the following chart:

RECTANGLE	AREA	PERIMETER
12 x 3	36 cm^2	30 cm

7. Have the learners construct as many rectangles as they can with an area of 36 cm^2 and record their findings on the chart.

RECTANGLE	AREA	PERIMETER
12 x 3	36 cm^2	30 cm
6 x 6	36 cm^2	24 cm
1 x 36	36 cm^2	74 cm
2 x 18	36 cm^2	40 cm
4 x 9	36 cm^2	26 cm

   "Which rectangle has the largest perimeter?"
   "Which rectangle has the smallest perimeter?"
   "When might it be useful to know how to find the smallest perimeter for a given area?"
   (One possibility is when making a garden that requires a boundary fence or wooden border. A square garden will require the smallest amount of fencing or border material.)

Follow-up activity

Ask the learners to work in pairs to find the area and perimeter of the rectangular faces of a variety of objects (books, shoe box, room, desk top, computer screen).

Fixed perimeter rectangles and area

Measurement progression, 5th step

The purpose of the activity

In this activity, the learners develop an understanding of how to calculate the area and perimeter of rectangles from the measurement of side lengths. They learn that the areas of rectangles with fixed perimeters do not remain the same.

The teaching points

- The learners understand that the length around an object is its perimeter.
- The learners understand that the perimeter of a rectangle is calculated by adding the lengths of four sides together or by adding the lengths of two adjacent sides together and multiplying the result by two.
- The learners understand that the area of a rectangle is calculated by multiplying together adjacent side lengths.
- The learners understand that the rectangle with the largest area for a fixed perimeter is a square.
- The learners can measure length using metres, centimetres and millimetres, and area using square centimetres (cm^2) and square millimeters (mm^2).
- Discuss with the learners relevant or authentic situations where measuring the perimeter or area of rectangles is applicable.

Resources

- Rulers and measuring tapes (marked in metres, centimetres, millimetres).
- Grid paper.
- Scissors.
- A selection of rectangular and square objects.

The guided teaching and learning sequence

1. Draw a rectangle on the board and ask the learners to identify the perimeter.

   "Which is the perimeter?"
   "How would you calculate the perimeter?"

2. Ask for a volunteer to measure the length of sides and record their measurements on the board. Use this as an opportunity to check the learners are measuring accurately and giving the measurement unit.

3. Ask the learners to calculate the perimeter from the side length measurements given.

   "How did you calculate the perimeter in centimetres?" (Use this as an opportunity to share addition strategies).

4. Give each of the learners some grid paper and ask them to draw rectangles, with whole number sides, that have perimeters of 24 centimetres.

5. Ask: "How many rectangles have you found?" (6)

6. Record the rectangles in a table.

RECTANGLE	PERIMETER	
11 x 1	24 cm	
9 x 3	24 cm	
4 x 8	24 cm	
7 x 5	24 cm	
2 x 10	24 cm	
6 x 6	24 cm	

"How do you know that you have found all the possible rectangles?" (Notice if the learners are systematic in their exploration. For example, if they start with the smallest side (1 centimetre) or the largest side (11 centimetres) and increase or decrease in ones from that side length until they get to side lengths of 6, etc) "Is a 10 x 2 rectangle the same as a 2 x 10 rectangle?" "Why?"

7. Ask the learners to explain how to calculate the area of a 10 x 2 rectangle.

   If they counted squares, ask if they could do it another way. Encourage them to notice that you multiply the side lengths together, which in this case is 10 x 2 = 20 cm^2.

8. Ask:

   "Do you think the rectangles with a perimeter of 24 centimetres have the same area?"
   "Which do you think will have the largest area?" "Why?"
   "Which will have the smallest area?" "Why?"

9. Ask the learners to complete the table to find out which has the smallest and largest area.

RECTANGLE	PERIMETER	AREA
11 x 1	24 cm	11 cm^2
9 x 3	24 cm	27 cm^2
4 x 8	24 cm	32 cm^2
7 x 5	24 cm	35 cm^2
2 x 10	24 cm	20 cm^2
6 x 6	24 cm	36 cm^2

Follow-up activity

Ask the learners to work in pairs to find the area and perimeter of the rectangular faces of a variety of objects (books, shoe box, room, desk top, computer screen).

Circumferences

Measurement progression, 5th step

The purpose of the activity

In this activity, the learners develop an understanding of how to calculate the circumference of circles and cylinders by using the radius or diameter measure.

The teaching points

- The learners understand that the circumference of a circle can be calculated by multiplying the diameter by pi (π) or by multiplying the radius by two times pi (π).
- The learners understand the need for calculating the circumference indirectly from measurements of the radius and diameter rather than measuring directly.
- The learners know that '3.14' (or 3) is an approximation for pi (π).
- The learners can measure length using millimetres.
- Discuss with the learners relevant or authentic situations where it is necessary to understand how to calculate the circumference.

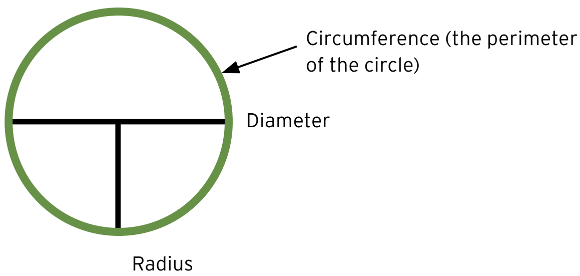
Resources

- String.
- Rulers and measuring tapes (marked in millimetres).
- A selection of cylinders.
- Compasses and pencils.
- Adhesive tape.

The guided teaching and learning sequence

1. Begin the session by asking the learners to use a compass to draw the following circles:
   - the smallest circle that can be drawn
   - the largest circle
   - four circles of different size that have the same centre point.
2. Discuss the features of a circle with the learners.

   "Where is the diameter of a circle?" "Does it always go through the centre point?"
   "Where is the radius?" "Does it always have one end on the centre point?"
   "How could you find the diameter if the centre point wasn't marked?" (Fold the circle in half.)
   "How could you find the radius if the centre point wasn't marked?" (Fold the circle in quarters.)



3. Ask the learners to work in pairs to collect together at least five circles or cylinders of different sizes (this could include the circles drawn previously). Ask them to measure the radius, the diameter and the circumference of each circle as accurately as they can and record their measurements in millimetres on a chart. Provide them with a collection of measuring tapes, rulers, string and tape.

CIRCLE	RADIUS (MILLIMETRES)	DIAMETER (MILLIMETRES)	CIRCUMFERENCE (MILLIMETRES)
1			
2			
3			
4			
5			

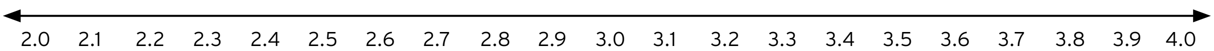
4. If the learners have difficulty measuring the circumference, you may want to discuss their problems and their solutions. One way to measure the circumference is to wrap string (which is flexible) around the outer edge of the circle, using tape to hold the string in place and then removing the string to measure it against a ruler.

5. After the learners have recorded the measurements of several circles on the chart, challenge them to think about the relationship between the diameter and the circumference.

   "Can you see a relationship between the lengths of the diameter and circumference?"

   You may need to prompt further by encouraging the learners to use a calculator to divide each of the circumferences by the diameter and notice that the result is close to 3.

6. Ask the learners to divide each of their circumferences by the diameter and to record their outcomes on a 'class' number line marked in tenths from 2 to 4.



7. If the measurements have been reasonably accurate, the outcomes should be clustered around 3.1 and 3.2, and the learners will be able to 'see' the relationship.

8. Write the formula $C = \pi d$ on the board. Ask:

   "What do you think the symbols are?"

continued...

9. After the learners have given you their ideas, you may need to tell them that the symbol is read as 'pi' and that pi has the value of approximately 3.14.

10. Write the formula $C = 2\pi r$ on the board.

    "What might this be?" "Will you get the same answer as $C = \pi d$?" "Why?" Have the learners share their solutions.

Follow-up activity

Follow up by posing problems that require the learners to apply the formula. Conclude by posing a problem that requires the learners to apply the formula $C = \pi d$.

1. The largest Ferris wheel in the world is in Yokohama City, Japan. It is 100 metres tall. About how far do you ride when you go around this Ferris wheel once?
   (C = 100 x 3.14, so one turn is approximately 314 metres.)

2. A large pizza has a circumference of 100 centimetres. What is the side length of the smallest box capable of holding the pizza?
   (100 = 3.14 x d so d = 100/3.14 = 31.8 centimetres. The side length of the box needs to be at least 31.4 centimetres to hold the large pizza.)

Areas of circles

Measurement progression, 5th step

The purpose of the activity

In this activity, the learners develop an understanding of how to calculate the area of a circle. This activity should follow the Circumferences activity above.

The teaching points

- The learners understand that the area of a circle can be calculated by multiplying the radius squared by pi (π).
- The learners are able to estimate the area of a circle.
- The learners can identify the diameter, radius and circumference of a circle.
- The learners know that the radius is half the diameter.
- The learners know that the circumference of a circle is approximately three times, and exactly pi (π) times, the diameter.
- The learners understand that r squared (r^2) means r x r.

Resource

- A variety of sizes of paper circles – a pair of each size.
- Scissors.

The guided teaching and learning sequence

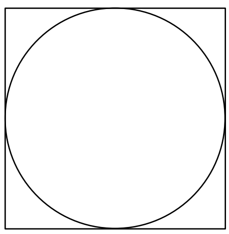
1. Ask the learners to discuss what they already know about the area of the circle. Record their responses on the board, including the formula $A = \pi r^2$ if it is given.
2. Give each pair of learners two paper circles of the same size and ask them to draw a diameter and a radius on one circle.
3. Discuss what they have done:

   "Does a diameter always pass through the centre point?" (yes)
   "Does the radius always end at the centre point?" (yes)
   "What is the relationship between the radius and the diameter?" (The diameter is twice the radius.)
   "Identify the circumference of the circle."
   "Is there a relationship between the diameter and the circumference?" (yes)
   "What is the relationship approximately and what symbol is used to describe it?" (The circumference is approximately three times the diameter and the symbol used to describe it is π.)
4. Ask the learners to discuss how they could estimate the area of their circle.

   You may need to remind them that the area is the number of square units needed to completely cover the circle.

   Methods of estimation could include:

   - estimating the number of square centimetres that would fit on the circle
   - estimating what fraction of a square metre the circle is
   - calculating the area of the square that the circle fits within, the side being the diameter of the circle, as an estimation of the area of the circle.



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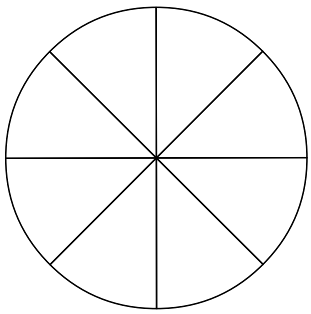
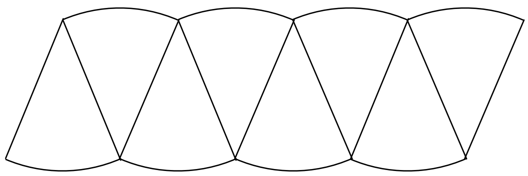
5. Ask the learners to estimate the area of their circle and record their estimation.
6. Explain to the learners that there is a formula for calculating the area of a circle and refer to it if it was given at the beginning of the session. Say that the learners are going to undertake an activity to justify the use of that formula to calculate the area of a circle. Ask them to fold the circle in half, in half again, and then in half again. When opened there will be 8 'sectors'. Ask the learners to cut out the sectors and reassemble them to form an approximate parallelogram. Draw the required shape on the board. Alternatively you may demonstrate the process.





7. Ask the learners to compare their first circle with the parallelogram and ask:

   "Is the area of the (approximate) parallelogram the same as the area of the circle?" (yes)
   "How would you find the area of the parallelogram?" (Multiply the base by the height at right angles between them.)
   "What part of the circle is the height of your parallelogram?" (radius)
   "What part of the circle are the two long sides of your parallelogram?" (circumference)
   "What part of the circle is one of the long sides of the parallelogram?" (half the circumference)
   "If the full circumference is $2\pi r$, what is half the circumference?" (πr)
   "If the area of the parallelogram is found by multiplying the base by height, write this using π and r." (π x r x r)
   "In what other ways can you write r x r?" (r^2)
   "Do you accept that you can use A= π x r^2 to find the area of a circle?"

8. Write the formula for the area of a circle $A = \pi r^2$ and ask the learners to use this formula to calculate the area of their circle.

Follow-up activity

Pose problems that require the learners to use the formula discussed above.

Calculating volumes of regular 3D objects

Measurement progression, 6th step

In this activity, the learners develop the understanding that the volume of regular 3D objects (for example, shoe boxes and cylinders) can be found by multiplying the area of the face by the height/length.

The teaching points

- The volume of regular 3D objects can be seen as being formed by the area of the face [rectangle for a shoebox, circle for a cylinder] being repeatedly 'stacked up'.

  This means that the volume of a regular 3D object can be found by calculating the area of the face and multiplying it by the height or length of the object. This principle means that it is possible to calculate the **volume** of a variety of regular 3D objects without learning lots of different formulae.

Resources

- A shoebox and a cardboard cylinder with the face cut out.
- A variety of regular 3D objects, including those with triangle and pentagon faces – fancy chocolate containers are good for this!

The guided teaching and learning sequence

1. Ask the learners to identify the shape of the face of the shoebox (rectangle) and cylinder (circle).
2. For both the shoebox and the cylinder, push the face through the object and suggest that the object is formed by the face being repeatedly 'stacked'. Ask the learners how this could help with calculating the volume of the shoebox and the cylinder.

   Listen and encourage the idea that if you know the area of the face of an object, you could find the volume by multiplying the area of the face by the height or length.
3. Ask for a volunteer to measure the length of the sides of the shoebox and record the measurements on a diagram on the board.
4. Ask:

   "If the volume of the shoebox can be found by multiplying the area of the face by the height or length, what face would you use to find the volume of the shoebox?"

   Further questions could include:

   "How many faces are there?" "Are they all different?" "Does it matter which one you use to calculate the volume?"
5. To find out whether it matters which face you use, divide the learners into three groups and allocate one face to each group. Ask each group to calculate the volume of the shoebox by finding the area of the face and multiplying it by the height or length.

   For example: Shoebox dimensions:
   25 centimetres x 15 centimetres x 10 centimetres
   Face 1: 25 centimetres x 15 centimetres = 375 cm^2
   Volume = 375 cm^2 x 10 centimetres = 3,750cm^3
   Face 2: 10 centimetres x 15 centimetres = 150 cm^2
   Volume = 150 cm^2 x 25 centimetres = 3,750 cm^3
   Face 3: 25 centimetres x 10 centimetres = 250 cm^2
   Volume = 250 cm^2 x 15 centimetres = 3,750 cm^3
6. Record each group's results exactly as they are given. For example, if you are told "3,750", record it as such. Ask whether this is a correct indication of the volume. Listen for and reinforce the idea that it is not because, without the unit of measurement, there is no indication of the size.

continued...

7. Ask:

   "What is the correct unit of measurement for the volume of the shoebox?"
   "Where does it come from?"

   Listen for and emphasise the idea that cm^3 comes from centimetres (the length or height) multiplied by cm^2 (the face area). You may need to revise the convention cm x cm x cm = cm^3.

8. Ask:

   "Does the calculated volume seem reasonable – would approximately 3,000 cubic centimetres fit in the box?"
   "Does it matter which face you use to find the volume?"

9. Restate the principle that the volume was calculated by multiplying the area of the face by the height or length and ask how this would apply to the cylinder.

Follow-up activity

Ask the learners to calculate the volumes of the cylinder and other regular 3D containers.

Appendices

Appendix A: Conversion dominoes (length)

50 cm	1,000 m	100 cm	0.5 cm	0.000005 km	200,000 cm
2 km	10 mm	0.000001 km	2,000 mm	0.1 cm	1,000 mm
0.0005 km	0.001 km	0.01 m	2,000 m	20,000 mm	1,000,000 mm
0.5 m	2,000,000 mm	500 mm	0.002 km	0.005 m	2,000 cm
0.00001 km	100,000 cm	0.001 m	0.02 km	2 m	5 mm
1 km	1 m	1 cm	200 cm	20 m	1 mm
50 cm	0.5 cm	10 mm	0.00001 km	100,000 cm	1 mm
0.02 km	1 m	2,000,000 mm	1,000,000 mm	0.000001 km	2,000 cm

2,000 m	1 km	2 km	0.1 cm
5 mm	0.001 km	1,000 mm	20,000 mm
0.0005 km	200 cm	2,000 mm	2 m
0.001 m	0.000005 km	0.005 m	20 m
100 cm	0.002 km	1 cm	200,000 cm
500 mm	0.5 m	1,000 m	0.01 m

Appendix B: Conversion dominoes (weight)

50 g	1,000 kg	100 g	0.5 g	0.000005 t	200,000 g
2 t	10 mg	0.000001 t	2,000 mg	0.1 g	1,000 mg
0.0005 t	0.001 t	0.01 kg	2,000 kg	20,000 mg	1,000,000 mg
0.5 kg	2,000,000 mg	500 mg	0.002 t	0.005 kg	2,000 g
0.00001 t	100,000 g	0.001 kg	0.02 t	2 kg	5 mg
1 t	1 kg	1 g	200 g	20 kg	1 mg
50 g	0.5 g	10 mg	0.00001 t	100,000 g	1 mg
0.02 t	1 kg	2,000,000 mg	1,000,000 mg	0.000001 t	2,000 g

2,000 kg	5 mg	0.0005 t	0.001 kg	100 g	500 mmg
1 t	0.001 t	200 g	0.000005 t	0.002 t	0.5 kg
2 t	1,000 mg	2,000 mg	0.005 kg	1 g	1,000 kg
0.1 g	20,000 mg	2 kg	20 kg	200,000 g	0.01 kg

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